Friday-5 AKT on 140207: d^{-1}, gauge fixing

January-28-14 9:15 AM

Claim $\langle \gamma_1 | d^{-1} | \gamma_2 \rangle = \ell(\gamma_1, \gamma_2)$

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claim $\alpha \in \Omega^2(\mathbb{R}^3)$ & $d\alpha = 0$

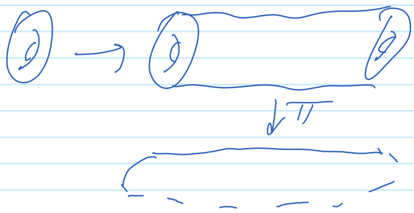
$$\begin{array}{ccc} \phi: \mathbb{R}^3_x \times \mathbb{R}^3_y & \longrightarrow & S^2, w \\ \pi_x \swarrow \qquad \searrow \pi_y & & \\ \mathbb{R}^3_x \qquad \mathbb{R}^3_y & & \end{array}$$

$$\sigma = \int_{\mathbb{R}^3_y} \left(\phi^*(w) \wedge \pi_y^* \alpha\right) \in \Omega^1(\mathbb{R}^3_x)$$

"a formula for d^{-1}"

Then $d\sigma = \alpha$

$$\begin{array}{ccc} F^k & \longrightarrow & T \\ & & \downarrow \pi \\ & & M \end{array}$$



Claim/def There is a unique $\pi_*: \Omega^n(T) \to \Omega^{n-k}(M)$ s.t. for every $w \in \Omega^n(T)$ & every $(n-k)$-cycle C in M,

$$\int_C \pi_*(w) = \int_{\pi^{-1}(C)} w$$

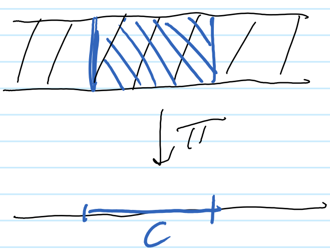
start line

Thm ("Stokes' for pushforwards")

with
$$\begin{array}{ccc} \partial F & \longrightarrow & T' \\ & & \downarrow \partial\pi \\ & & M \end{array}, \qquad d\pi_* w = \pi_* dw - (\partial\pi)_* w$$

Post Morton at bottom.

Pf



$w \in \Omega^n(T)$

C a $(n-k+1)$-cycle

Easy property:
$$\begin{array}{ccc} F^k & \longrightarrow & T \quad w \\ & & \pi \downarrow \\ & & M \quad \alpha \end{array} \qquad \pi_*(w \wedge \pi^* \alpha) = (\pi_* w) \wedge \alpha$$

Proof of Claim $\mathbb{R}^3 \setminus pt \longrightarrow \tilde{C}_2(\mathbb{R}^3) \xrightarrow{\phi} S^2, w$

Proof of Claim

$$\begin{tikzcd} \mathbb{R}^3 \setminus pt \arrow[r] & \tilde{C}_2(\mathbb{R}^3) \arrow[r, "\phi"] \arrow[d, "\pi_x"] \arrow[dr, "\pi_y"] & S^2, w \\ & \mathbb{R}^3_x & \mathbb{R}^3_y, \alpha \end{tikzcd}$$

$\sigma = \pi_{x*}\left((\phi^* w) \wedge \pi_y^*(\alpha)\right)$

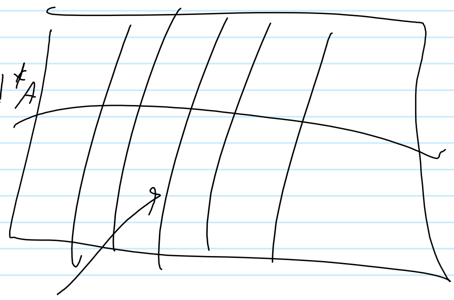
satisfies $-d\sigma = \alpha$, if $d\alpha = 0$

Pf $d\sigma = \pi_{x*}(0) - (\partial\pi_x)_*(\phi^* w \wedge \pi_y^* \alpha)$

$= -(\partial\pi_x)_*(\phi^* w) \wedge \alpha = -\alpha$

"Gauge Fixing"

$$Z(\gamma_1,\gamma_2) = \int \mathcal{D}A \, e^{\frac{i}{4\pi}\int_{\mathbb{R}^3} A\wedge dA} \int_{\gamma_1} A \int_{\gamma_2} A$$



$\delta(x) = \int e^{ixy}\, dy$

$\delta(d^*A) = \int \mathcal{D}\phi \, e^{i2\int \phi\, d^*A}$ $\quad \phi \in \Omega^3$

$$Z \mapsto \int \mathcal{D}A\mathcal{D}\phi \, e^{i\int (A\wedge dA + 2\phi\, d^*A)}$$

$$\int A\wedge dA + 2\phi\, d^*A = \left\langle \begin{pmatrix} A \\ \phi \end{pmatrix}, \underbrace{\begin{pmatrix} *d & d* \\ d* & 0 \end{pmatrix}}_{L^-} \begin{pmatrix} A \\ \phi \end{pmatrix} \right\rangle$$

in $\Omega^1 \times \Omega^3$

Of all $\{A + df\}$, which has the minimal L^2-norm?

$A \perp df \quad \forall f$

$0 = \langle A, df \rangle = \langle d^*A, f \rangle$

$\Rightarrow d^*A = 0$

done live

What's d^*?

"div", $*d*$

$(L^-)^2 = \pm\Delta \qquad (L^-)^{-1} = L^-(\Delta^{-1}) = \ldots$

Then Feynman diagrams in $\mathbb{R}^n$

or

Non-Abelian Chern-Simons

Post Mortem: Further properties of pushforwards:

1. Push forward of a pushforward is a pushforward.
2. Pushforward commutes w/ pullback:

$$\begin{tikzcd} T_2 = \beta^* T_1 \arrow[r, "\beta"] \arrow[d, "\pi_2"'] & T_1, \omega \arrow[d, "\pi_1"] \\ B_2 \arrow[r, "\beta"'] & B_1 \end{tikzcd}$$

$$\beta^* \pi_{1*} \omega = \pi_{2*} \beta^* \omega$$